we can WORK it out

Use Mike Askew's advice on setting up models and help children make sense of maths problems...

Which of these problems do you think 8-year-old children find easier?

Mr Chang bought some DVDs. He bought DVDs costing £7 each. He spent £42 pounds. How many DVDs did he buy?

Mrs Chang bought some CDs. She bought 5 CDs each costing the same amount. She spent £35 pounds. How much did each CD cost?

Most teachers I've asked think the second is probably easier as children are more likely to know that 7 times 5 is 35 than 6 times 7 is 42. In practice, most children find the first problem easier. However, this is not due to the numbers involved, but because of how they set about solving it.

The children I've posed these problems to have few difficulties with the first problem. Generally they count on in sevens, putting out one finger for each count, until they reach 42. Seeing that they have put out six fingers they know that this models the number of DVDs.

In contrast, unless really confident, many children approach the second problem using trial and improvement: guessing what the price of one CD might be (say, £6) and then using repeated addition to find the price of five. Depending on whether or not that total is greater or less than £35 they adjust their estimate, recalculate and continue in this fashion until they 'hit' £35.

Part of the reason why the Mr Chang problem is easier than the Mrs Chang one may be the familiarity and 'realness' of the context. When we go shopping, we usually know the price of each item; it is rare that we need to find the price of an individual item given a total spend. But the different structures to the problem also affect the level of difficulty.

Into the unknown

Last month, I wrote about the work of the Cognitively Guided Instruction project, which provides a framework for the different types of 'root' problems; for example, addition and subtraction arising from 'join and separate', 'compare' and 'part-part-whole' situations. These writers go on to show how the different root situations can give rise to further categories of problems depending on the position of the 'unknown' in the 'story'. For example, within join problems we can have:

End unknown: Four frogs were down a well when three more jumped in.
Change unknown: In the morning there were four frogs in the well. By the afternoon there were seven.
Start unknown: Three more frogs jumped in and there were then seven.

Returning to Mr Chang and the DVDs, if you strip the essence of the problem away from the context it becomes 'what do I have to multiply 7 by to get 42? This can be expressed mathematically as:

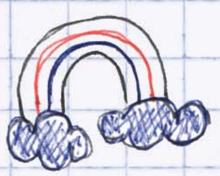
$7 \times ? = 42$

This is the mathematical model that children implicitly work with when counting on in sevens to 42. And one they find easy to use by modelling with fingers.

Turning to Mrs Chang, the essence of her problem is what number do I have to multiply by 5 to get 35? Symbolically:

$? \times 5 = 35$

This is more difficult to represent in the physical world – you have to guess at what goes in the box, hence the children's difficulty with this problem.



Time 4 T

Put this t-table on the board about how much sugar Bob likes in his tea.

Cups of tea	*Sugar lumps*
1	*3*
5	*15*

Talk with the children about what information the table shows. Cover up each of the numbers in turn. Can the children come up with a simple problem that has the covered number as the answer. For example, covering up the 5:

Bob likes 3 lumps of sugar in a cup of tea. If Bob uses 15 lumps of sugar in a day, how many cups of tea does he drink?

Modelling multiplication and division problems

Setting up a numerical sentence (equation), like those above, can help children to understand the mathematical calculation they need to carry out. But the difficulty with this is that, in order to write the equation, you already need to have figured out what you are being asked to do.

However, putting the information in a simple t-table can help children to make sense of a problem. And the simplest way to introduce this is through a standard multiplication problem. Let's go back to our CD store.

Wok buys 6 DVDs. They are in the sale and cost £5 each. How much does Wok spend?

The mathematical expression is easy to set up here:

$6 \times 5 = 30$

How many numbers are involved in this problem? If you think this is a trick question, you are right. The obvious answer is three: 6, 5, and 30. But there is a hidden number in the question and it is the 'one' hidden in each:

Wok buys 6 DVDs. They are in the sale and 1 DVD costs £5. How much does Wok spend?

We can make this hidden 'one' explicit by putting the information in a t-table.

DVDs	*Cost*
1	*5*
6	*[ ]*

The first advantage of setting the problem out like this is that you can figure out the answer in different ways. For example, reading down the table, the 1 has become six times bigger, and so the 5 also has to be multiplied by 6. Or, reading across the rows, the 1 has become 5 times bigger and so the six also becomes 5 times bigger.

While setting out a simple calculation in this form may feel like using the proverbial sledgehammer, the advantages start to become clearer when working with larger numbers. Suppose I want the cost of 15 DVDs at £8.50 each and haven't yet learned to multiply decimals. A t-table can help me find the answer strategically by calculating 'partial products'.

DVDs	*Cost*
1	*8.50*
10	*85.00*
(multiply £8.50 by 10)	
5	*42.50*
(halve the cost of 10)	
15	*127.50*
(add the cost of 10 and 5)	

It is with division problems that the t-table starts to be really helpful. Let's put Mr Chang's information into one:

DVDs	*Cost*
1	*7*
[ ]	*42*

I can see that I need to figure out the missing number of DVDs. And I can do this in a variety of ways. I can note that the 1 has scaled up to 7 and ask myself what number multiplied by 7 gives 42. Or I can read down the columns: what must 7 be multiplied by to get 42? The 1 must be multiplied by the same number. If I'm not that confident I could fill in some intermediate values to help me build up to 42:

DVDs	*Cost*
1	*7*
2	*14*
4	*28*
[6]	*42*
(add the price for 2 and 4 CDs)	

Mrs Chang's shopping trip looks like this:

CDs	*Cost*
1	*[ ]*
5	*35*

Again, relationships can be explored by thinking across the rows (what was the five multiplied by to get 35?) or up the columns (5 is divided by 5 to get to 1, so divide 35 by 5).

But perhaps the greatest 'payoff' to using the t-table is the clear and explicit link between multiplication and division problems and how closely they are related. Happy modelling!

Mike Askew is Director of BEAM Education and Professor of Maths Education, King's College, London. BEAM Education is a specialist publisher of mathematical books and resources, and provides training consultancy in mathematics education. They publish a range of more than 100 books, mathematical games and equipment.

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